Chapter II

1)

$$V = L\frac{dI}{dt} + RI + \frac{1}{C}\int Idt$$

Applying the transform, considering a DC voltage source and a discharged system:

$$\frac{V}{s} = sLI + RI + \frac{I}{sC} \Leftrightarrow I = \frac{V}{s}\frac{1}{sL + R + \frac{1}{sC}} \Leftrightarrow I = \frac{V}{s^2 L + sR + \frac{1}{C}} \Leftrightarrow I = \frac{\frac{V}{L}}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

The roots of the denominator are calculated:

$$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$$

Notice that the roots are identical to those obtained in equation 2.52 in chapter 2. Thus, we can rewrite the expression as:

$$I = \frac{\frac{V}{L}}{(s - s_1)(s - s_2)}$$

Applying the method of partial fractions is obtained:

$$\frac{1}{(s - s_1)(s - s_2)} = \frac{A}{(s - s_1)} + \frac{B}{(s - s_2)} \Rightarrow A(s - s_2) + B(s - s_1) = 1$$

Thus:

$$A = -B \wedge A(s_1 - s_2) = 1 \Leftrightarrow A = \frac{1}{2\sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}}$$

We can then write:

$$I = \frac{V}{L}\left(\frac{\frac{1}{2\sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}}}{s - \left(-\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}\right)} - \frac{\frac{1}{2\sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}}}{s - \left(-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}\right)}\right)$$

The application of the Laplace transform is now easy and the expression obtain will be identical to the one shown in equation 2.53.

Note: In the book we did not developed the expression this much. However, the resolution of 2.54 would give the same. As a test, try to compare the values of this expression with those of the example.

2)

Note: This is only a theoretical exercise made in order to give a gasp on the concept of the Laplace transform. The system is much easier to analyse in the time domain.

$$I = I_R + I_L + I_C \Leftrightarrow I = \frac{V}{R} + \frac{1}{L}\int V dt + C\frac{dV}{dt}$$

Appling the Laplace transform and considering that the load was discharged prior to the connection.

$$I = \frac{V}{s}\frac{1}{R} + \frac{V}{s}\frac{1}{sL} + \frac{V}{s}sC \Leftrightarrow I = \frac{V}{sR} + \frac{V}{s^2 L} + VC$$

The inverse Laplace transform can be applied to any of the terms independently and it is obtained the current in function of the time.

$$I(t) = \frac{V}{R} + \frac{V}{L}t + VC\delta(t)$$

The current is each of the three elements is independent of each other. The current in the resistor is constant, while the current in the inductor increases linearly with the time (an inductor is similar, but not alike, to a short-circuit for a DC voltage) and the current in the capacitor is a Dirac impulse at the switching instant whose magnitude is proportional to the voltage and capacitance. The latter current is only theoretical as such impulse would be impossible in reality.

The solving of the system for a DC current was only a theoretical exercise, because of the capacitor. However, it is possible to have in reality a similar, but not alike, system connected to an AC voltage source.

$$I = \frac{V\sin(\omega t)}{R} + \frac{1}{L}\int V\sin(\omega t)dt + C\frac{d(V\sin(\omega t))}{dt} \Leftrightarrow I = \frac{V\sin(\omega t)}{R} - \frac{V}{\omega L}\cos(\omega t) + K + \omega CV\cos(\omega t)$$

The term *K* is a constant present in order to maintain the continuity of the current in the inductor. In this case it is equal to *V/(ωL)*.

We can see that the final result is easily obtained by just doing the derivative, but we will do the Laplace transform in order to train.

$$I = \frac{V\sin(\omega t)}{R} + \frac{1}{L}\int V\sin(\omega t)dt + C\frac{d(V\sin(\omega t))}{dt} \Leftrightarrow I = \frac{V}{R}\frac{\omega}{s^2+\omega^2} + \frac{V}{L}\frac{\omega}{s(s^2+\omega^2)} + sCV\frac{\omega}{s^2+\omega^2}$$

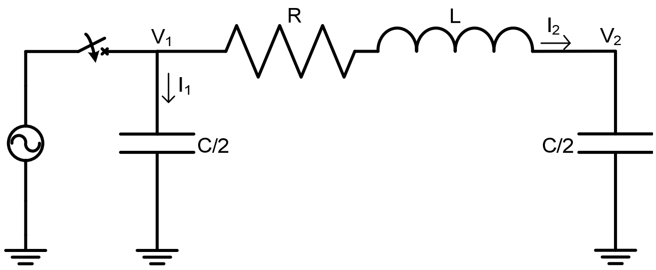
$$\Leftrightarrow I = \frac{V}{R}\frac{\omega}{s^2+\omega^2} + \frac{V\omega}{L}\left(\frac{1}{s\omega^2} - \frac{s}{\omega^2(s^2+\omega^2)}\right) + \omega CV\frac{s}{s^2+\omega^2}$$

Inverting the system to the time domain we obtain the same result from before.

$$I = \frac{V\sin(\omega t)}{R} + \frac{V}{\omega L} - \frac{V}{\omega L}\cos(\omega t) + \omega CV\cos(\omega t)$$

3)

The objective is to calculate V_2 for the circuit below on the first instants of the energisation.



We start by calculating the current I_2, which is given by the expression used for the calculation of a current in a RLC load.

Thus, we start by calculating the roots as shown in 2.52.

$$\lambda_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{L\frac{C}{2}}} \Leftrightarrow \lambda_{1,2} = -\frac{0.62}{2\cdot 0.0447} \pm \sqrt{\left(\frac{0.62}{2\cdot 0.0447}\right)^2 - \frac{1}{0.0447\cdot 1.95\times 10^{-6}}} \Leftrightarrow \lambda_{1,2} = -6.94 \pm j3387$$

The next step is to handle the system initial conditions for the forced and homogeneous regimes:

$$I_{2,f}(0) = \frac{V_P}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega\frac{C}{2}}\right)^2}} \sin\left(0 + \theta - \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega\frac{C}{2}}}{R}\right)\right) \Leftrightarrow I_{2,f}(0) \simeq \frac{V_P}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega\frac{C}{2}}\right)^2}} \sin\left(0 + \frac{\pi}{2} + \frac{\pi}{2}\right) \Leftrightarrow I_{2,f}(0) \simeq 0A$$

$$\dot{I}_{2,h}(0) = \left(\frac{V_P\omega}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega\frac{C}{2}}\right)^2}} \cos\left(\theta - \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)\right)\right) - \left(\frac{V_P}{L}\sin(\theta)\right)$$

$$\Leftrightarrow \dot{I}_{2,h}(0) = \left(\frac{100\times 10^3 \cdot 2\pi 50}{\sqrt{0.62^2 + \left(2\pi 50\cdot 0.0447 - \frac{1}{2\pi 50\cdot 1.95\times 10^{-6}}\right)^2}} \cos\left(\frac{\pi}{2} - \tan^{-1}\left(\frac{2\pi 50\cdot 0.0447 - \frac{1}{2\pi 50\cdot 1.95\times 10^{-6}}}{0.62}\right)\right)\right) - \left(\frac{100\times 10^3}{0.0447}\sin\left(\frac{\pi}{2}\right)\right)$$

$$\Leftrightarrow \dot{I}_{2,h}(0) = \left(19.4\times 10^3 \cos\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right)\right) - 2237\times 10^3 \Leftrightarrow \dot{I}_{2,h}(0) = -2256\times 10^3 A$$

We can now write the final equation for the current.

$$I_2(t)=\frac{V_P}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega\frac{C}{2}}\right)^2}}\sin\left(\omega t+\theta-\tan^{-1}\left(\frac{\omega L-\frac{1}{\omega\frac{C}{2}}}{R}\right)\right)+\frac{\dot{I}_{2,h}(0)+I_{2,f}(0)\lambda_2}{\lambda_1-\lambda_2}e^{\lambda_1 t}+\frac{\dot{I}_h(0)+I_{2,f}(0)\lambda_1}{\lambda_2-\lambda_1}e^{\lambda_2 t}$$

$$\Leftrightarrow I_2(t)=61.75\sin\left(\omega t+\frac{\pi}{2}+\frac{\pi}{2}\right)+\frac{-2256\times10^3-0\cdot(-6.94-j3387)}{j6774}e^{(-6.94+j3387)t}+\frac{-2256\times10^3-0\cdot(-6.94+j3387)}{-j6774}e^{(-6.94-j3387)t}$$

$$\Leftrightarrow I_2(t)=-61.75\sin(\omega t)+e^{-6.94t}\left(\frac{-2256\times10^3}{j6774}\left(e^{j3387t}-e^{-j3387t}\right)\right)\Leftrightarrow I_2(t)=-61.25\sin(\omega t)-e^{-6.94t}\left(\frac{333}{j}\left(e^{j3387t}-e^{-j3387t}\right)\right)$$

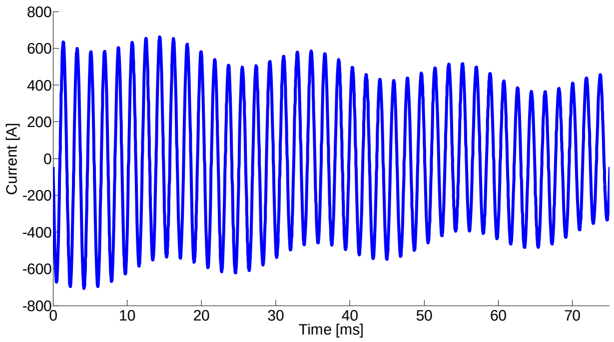
$$\Leftrightarrow I_2(t)=-61.75\sin(\omega t)-e^{-6.94t}\left(666\sin(3387t)\right)$$

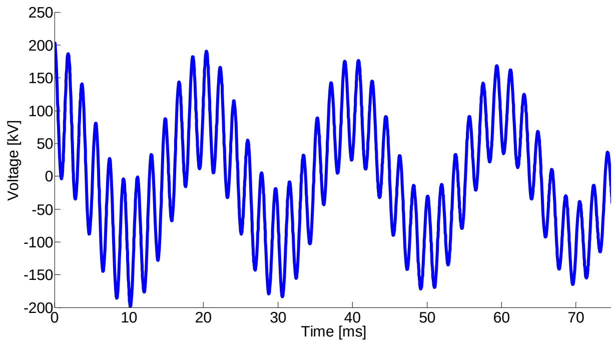
The voltage is now calculated by doing the integral (solved numerically):

$$V_2=\frac{1}{\frac{C}{2}}\int I_2(t)\,dt\Leftrightarrow V_2=\frac{1}{1.95\times10^{-6}}\int\left(-61.75\sin(\omega t)-e^{-6.94t}\left(666\sin(3387t)\right)\right)dt$$

$$\Leftrightarrow V_2\simeq\frac{1}{1.95\times10^{-6}}\left(\frac{61.75}{\omega}\cos(\omega t)-666\frac{e^{-6.94t}}{(-6.94)^2+3387^2}\left(-6.94\sin(3387t)-3387\cos(3387t)\right)\right)$$

$$\Leftrightarrow V2\simeq\frac{1}{1.95\times10^{-6}}\left(0.2\cos(\omega t)+e^{-6.94t}\left(4\times10^{-4}\sin(3387t)+0.2\cos(3387t)\right)\right)$$





4)

We already have the expression for the current I_2. Thus it is only necessary to calculate the expression to I_1 and add both.

$$V_1 = \frac{1}{\frac{C}{2}}\int I_1 dt \Leftrightarrow I_1 = 1.95\times10^{-6}\frac{d\left(100\times10^3\sin\left(\omega t+\frac{\pi}{2}\right)\right)}{dt} \Leftrightarrow I_1 = 1.95\times10^{-6}\cdot100\times10^3\omega\cos\left(\omega t+\frac{\pi}{2}\right)$$
$$\Leftrightarrow I_1 = -61.26\sin(\omega t)$$

Notice that the magnitude of the current is almost alike to the steady-state value of I_2. This is the result of the capacitance being dominant in a cable.
It is also important to notice that the current can change instantly at the switching instant, meaning that the derivative can be infinite. This is impossible in a real system, where the inductance will delay the current. However, in this simplified model the current I_1 does not have a transient.

Finally, we can write the current at the sending end:

$$I(t) = -61.26\sin(\omega t) - 61.75\sin(\omega t) - e^{-6.94t}\left(666\sin(3387t)\right)$$